IPM component 3 – STEM
On Relativism and absolutism

Introduction

In these notes we will address the aspect of whether there are such things as absolute facts (things that are absolutely true and will never change) or relative facts (things which are true only in certain circumstances but not true in other circumstances). Beyond this, is there such a thing as truth being relative to people, for example, "This is your truth, and this is my truth"? (the answer to this is no).

Exercises

Let us start with some exercises.

1) Consider the table of numbers below. Can you see what is happening? If so, would you say that the results of arithmetic are absolute or relative? If they are relative, what are they relative to?

1 + 0 = 0	1 + 1 = 10							
1 + 0 = 0	1 + 1 = 2	1 + 2 = 10						
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 10					
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 10				
….	….	….	…	…	…			
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7	1 + 7 = 10	
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7	1 + 7 = 8	1 + 8 = 10
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7	1 + 7 = 8	1 + 8 = 9

1 + 9 = 10

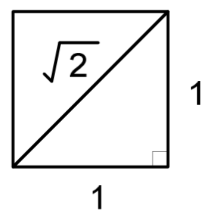
2) Is it true that all angles in a triangle add up to 180 degrees? If not, what is the sum of the angles of this different triangle?

Is it true that two parallel lines never intersect? If not, what ultimately happens to the direction of the two lines?

Example 1

In the days of the ancient Greeks the only numbers which existed (i.e. which were considered to be numbers) were integers. They could not conceive of rational fractions (i.e. numbers like p/q where p and q are integers), although they did have ratios of integers. What this means is that they were able to write things like $p\!:q$. They did not interpret this as a fraction, simply as a comparison of two distinct integers. So if they had two lines, one of length 2m and one of length 3m the ratio 2 : 3 represents a line of 2m being compared to a line of 3m (for us we would say the ratio 2 : 3 represents a line with one part being 2/5m long and the other part being 3/5 m long). And if they didn't conceive of rational fractions they certainly did not conceive of irrational numbers such as $\sqrt{2}$.

With this in mind consider the following diagram of a unit square (i.e. a square with sides 1)



It is clear geometrically that the diagonal exists. It can, in fact, be constructed using a straight-edge and compass (the only two instruments allowed by the Greeks for constructing geometric figures). However, the number $\sqrt{2}$ itself does not exist. The reason for this is that the Greek never accepted infinite processes into their mathematics. But the number $\sqrt{2} = 1.414213562 \ldots$ is a decimal number which has an infinite number of decimal digits. The arithmetic process for calculating $\sqrt{2}$ (by repeated subtraction or division) continues forever. So the number of decimal digits continues "expanding" forever. From this perspective of an ever expanding number of decimal digits $\sqrt{2}$ is not finite. Because of this the Greeks said that $\sqrt{2}$ could not be a number.

So we have a dichotomy: the diagonal line exists as a geometric object but the number does not exist. This is because the Greeks did accept the idea of irrationality (which they called incommensurability) but only for geometry not for arithmetic. This is an example of mathematical relativism in the sense that the Greeks accepted irrationality but only relative to geometry not numbers.

Example 2

Finally, at some point during the 16th and 17th centuries square roots became accepted. However, the same problem arose for imaginary numbers as it did for square roots. For example, as algebra became accepted as the main method of mathematical analysis so mathematicians ended up solving things such as

$$x^2 + 1 = 0.$$

Today we know the answer to this as i, the imaginary number, but in the days of the 16th and 17th and 18th centuries things like $\sqrt{-1}$ was considered ridiculous. Another way of expressing this result is as $a \times a = -1$, where a is some number. But people could not understand how two numbers multiplied together could produce a negative result. This is because mathematicians had not yet defined and classified the different types of numbers that we now know about. So $\sqrt{-1}$ could not exist.

Similarly for negative numbers. It was believed that negative numbers could not exists because you could not have a negative size or length or weight or area. So the concept of only positive numbers existing was relative to the physical world. Nowadays we know that negative numbers do not exists relative to N but they do exist relative to Z.

However, today we have a well defined system and hierarchy of numbers such that some numbers do exists in one domain but not in another domain, e.g.

- the number -1 does not exists in $\mathbb{N}$ but does exists in $\mathbb{Z}$;
- the number ½ does not exists in $\mathbb{Z}$ but does exists in $\mathbb{Q}$;
- the number $\sqrt{2}$ does not exists in $\mathbb{Q}$ but does exists in $\mathbb{R}$;
- the number $\sqrt{-1}$ does not exists in $\mathbb{R}$ but does exists in $\mathbb{C}$.

Example 3

Up until the mid to end 19th century all mathematicians believed that $a \times b = b \times a$ was always true for all numbers. Such a property of number is called *commutativity*. So this could be considered as an example of absolutism in maths.

However, as time and mathematics progressed there came to be found non-commutative algebra. In other words, arithmetic was found arithmetic where $a \times b \neq b \times a$. We now have simple examples of this: if A and B are two matrices then $A \times B \neq B \times A$. Also, in vectors the cross product of two vectors **a** and **b** is not commutative: $\boldsymbol{a} \times \boldsymbol{b} \neq \boldsymbol{b} \times \boldsymbol{a}$.

So we now have the situation where commutativity is not an absolute property of numbers or other mathematical objects. Not all arithmetic operations obey commutativity. The result of arithmetic operations is relative to how one defines the operations on numbers or vectors.

<u>Example 4: An example of mathematics based on one society's conception of fractions</u>

The ancient Egyptians used something called *unit fractions*. These are fractions in which the numerator is 1. For example, 1/5, 1/8, 1/127, etc. The ancient Egyptians refused to deal with any fractions that weren't unit fractions (with the sole exception of 2/3, which had its own special hieroglyph). Nobody knows exactly why they did this.

So instead of writing 2/5 they would write 1/3 + 1/15. Other examples include

$$2/7 = 1/4 + 1/28$$
$$2/13 = 1/8 + 1/52 + 1/104$$
$$2/15 = 1/10 + 1/30$$

Sometimes representing fractions this way makes it easier to tell when one is bigger than another. Is 55/84 larger than 7/11? The unit-fraction representation makes it clear that it is:

$$55/84 = 1/2 + 1/7 + 1/84$$
$$7/11 = 1/2 + 1/8 + 1/88.$$

The Egyptians made things more complicated by the fact that they did not allow fractions to be repeated. So you could not write things like 4/5 = 1/5 + 1/5 + 1/5 + 1/5. All the denominators had to be distinct. So the question is, Is it possible to express any ordinary fraction as a sum of unit fractions? Yes. I won't go through this here. If you are intersted in knowing more see p154 onwards of *The Man Who Loved Only Numbers: The story of Paul Erdos and the search for mathematical truth*, Paul Hoffman (1998), Fourth Estate publishing.

Does this way of looking at fractions make for arithmetic relativism? In other words, is the ancient Egyptian way of conceiving of fractions fundamentally different from our own? Yes. Does this mean that their underlying conception of what fractions really are, what their essence is, is relativistic? No. not in my opinion. Because although their's was not an efficient way of writing fractions, and the arithmetic would be more laborious, they still would need to understand the concept of "fraction-ness", i.e. the fundamental conception of parts-to-whole.

We now look at some examples where aspects of mathematics were defined on the basis of social influences.

Example 5: An example of mathematics based on one society's conception of numbers

Over the course of the 6^{th}, 5^{th}, and 4^{th} centuries B.C. Pythagoras and his followers came to believe that everything in the universe could be reduced to numbers and the study of numerical relationships. But more than this they believed that the only numbers which existed were what we today call the natural number ($\mathbb{N}$). This primacy of natural numbers over everything else was based on the mystical belief that numbers could not only describe nature but also life and social affairs. As W. R. Knorr [30] says "The Pythagoreans took numbers to be their universal principle". For example, in life and social affairs, the number 1 represented unity and the origin of all things, the number 2 represented the feminine, and the number 3 represented the masculine. The number 3 also represented the "whole" because it could be separated into three units representing "beginning–middle–end". The number 4 represented justice. The number 5 represented marriage because it was the result of summing what was considered as the first two numbers: 2 + 3 = 5, and so on.

Since their conception of number applied not only to maths but also to life one might say that this perspective on numbers is an absolutist perspective.

Example 6: An example of mathematics based on one society's aversion to infinity

The ancient Greeks had decided to never use infinity and the infinitely small in their mathematics because they didn't believe that infinite processes of arithmetic could produce finite answer. For example, 1/2 + 1/4 + 1/8 + ... is an infinite series involving terms which ultimately become infinitely small. The Greeks knew that this series equalled 1 because they could provide a real example of this, i.e. a string 1 metre long which is continually cut in half. But the process of forever cutting in half requires an infinite amount of time which is clearly not possible. This seems to create a contradiction: one can arrive at a finite answer by performing a process which requires an infinite amount of time! So it is that the Greeks decided to never use infinities and infinitesimals in their maths. This is the first example of mathematics being influenced because of the peoples' aversion to some aspect of arithmetic.

The second example is that this attitude of the Greeks persisted for 2000 years. By the 16th century the Jesuits (a religious order of the catholic church) completely banned the use of the infinitely small in mathematics. How could they do this? what did a religious order have to do with controlling the content of mathematics? Well, throughout the dark ages of Europe (and probably before) the only way to get an education was to become a priest or a monk. There were no secular schools. As Europe moved into the renaissance period the monasteries continued to be the only place where one could get an education. This was primarily religious, but the trainee monks would also learn philosophy, medicine, and later, mathematics (even Isaac Newton had to take some kind of religious vows before he was allowed to take up his university post as Cambridge professor). Rare was the educated person who had not been part of a monastic order. These people were self-taught, and this was no mean feat. So it was the Roman church who set the tone for what was morally acceptable or not in mathematics.

Hence, as part of their religious philosophy, the Jesuits had adopted Aristotle's views on the infinite and the infinitely divisible who had said that such things were impossible (there are some subtleties here which I won't go into). To the Jesuits Aristotle was the master, and was therefore infallible in the views he expounded. So, geometric things such as lines could not be infinitely thin. This attitude by the church forced one mathematician called Cavalieri (1598 – 1647) to come up with the idea of indivisible lines. these were lines which were as thin as they could be but no thinner. In other words, there was a limit to how thin lines could be, and they could not be thinner than this. This made lines finite in width. Such an idea then circumvented the issue of a line being infinitely thin.

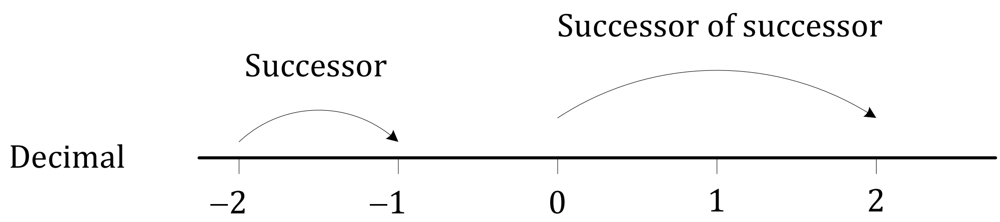
In all six examples above we see that it is peoples' beliefs (about mathematics or society) that hindered the development of mathematics.

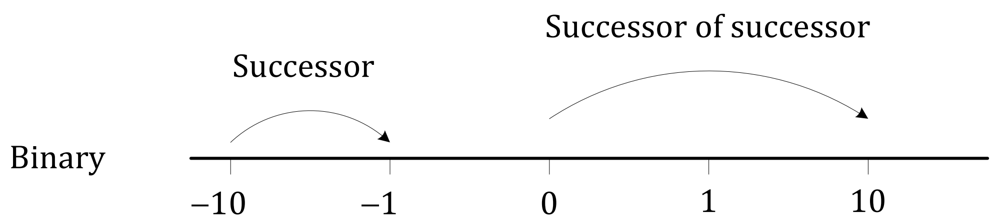
Definition

Absolutism

Absolutism is the position whereby some object or phenomenon is always true at all times whatever the historical, social or cultural context of the discovery of that object or phenomenon was discovered. For example, gravity is an absolute phenomenon. The effect or behaviour of gravity does not change depending on social, or historical or cultural context. Isaac Newton (1643 – 1727) is the first person to have developed a mathematical theory of gravity. He was an Englishman brought up during the 1600s in the social context of that time. But this does not mean that the theory he developed is a particularly renaissance English theory of gravity which would have been different if he had been a Chinese person living in the 4th century. So, someone else from Arabia or China or India or south-east Asia or south America would have had to develop a the same theory as Newton. The mathematical formulation of the theory may have differed (i.e. the algebraic expression may have been different) but the physical meaning of the mathematics would still have had to relate to the underlying physical phenomenon/effect of gravity.

Returning to exercise 1 above, where the results of arithmetic were different based on which number base we were using, we might think that this is an example of relativism in mathematics. My opinion is that this is not an example of relativism. Rather, this is an example of arithmetic being contingent on the number based used. There aren't different mathematical truths about arithmetic. It is just that there are different *representations* about an underlying truth of arithmetic. This underlying truth relates to addition and can be described as "successor to", as illustrated below.





(note that subtraction can be seen as addition of a negative, multiplication can be seen as addition, and division can be seen as subtraction).

The same applies to fractions. For example, we can write 1/3 or we can write 0.33333333... or we can show the geometric version as a line cut into three equal pieces. All of these are just different representations of the same fundamental absolute idea of "one-third". the concept of "one-third" is not relative to some social, historical context. And did you know that there are many ways to define a real number such as sqrt(2)? The two most well known ones are called Dedekind cuts and equivalence classes of Cauchy sequences. I won't go into details of what these mean, but it is just to show that real numbers are an absolute truth of mathematics even if there are many different ways to define them.

To see why this might be so, let us consider arithmetic, and therefore mathematics, being built on the binary system or on a hexadecimal (base 16) system. Whatever mathematics would look like through these number systems we would still be able to build bridges that stand, planes that fly, and ships that float using this mathematics. And even if aliens from another planet had a mathematics not constructed on geometry or numbers, but on something else we cannot conceive of, any bridges, planes and ships built on their mathematics would have to obey the same physical principles as ours. So mathematics references an absolute truth (for example the truth of the phenomena of gravity, aerodynamics, hydrostatic, chemical process, etc.), but mathematics itself may be relative, or better still, contingent in the sense illustrated in exercise 1.

Whatever form of symbolism, abstraction and generalisation mathematics takes, mathematics references a universal or absolute truth. We would still have come to the same results we have today. In other words, we would still have been able to build those bridges, planes etc., and study hermodynamics, chemistry, physics to obtain the same results and explanations of physical phenomena.

<u>*Relativism*</u>

The *co-variance* definition of relativism says that some object or phenomenon is dependent on the context or domain of applicability. For example, in exercise 1) above the results (the objects) of the arithmetic are relative to the number base (context or domain of applicability) we are using. Another example: two lines (the objects) that are parallel are always parallel is relative to Euclidean geometry (domain of applicability), since in other geometries (hyperbolic, elliptical, etc) this is not true. Another way of describing this kind of relativism is as *conceptual relativism*, where certain concepts (the results of arithmetic or parallel-ness) are true only with respect to a given frame of reference (numbers bases or types of geometries).

Another way to define relativism is by contrasting it with concepts which it is not (this defines relativism in a negative way). So adopting this idea of *relativism by contrast* we might say that relativism is not absolutism. For example, the principle that $a \times b = b \times a$ for all numbers cannot be absolutely true since we now know that $A \times B \neq B \times A$ where A and B are matrices. In mathematics this is called a counter example. Another example is "if you multiply two numbers you always get a bigger number". This is not true since we know that $-2 \times 3 = -6$, and -6 is smaller than -2 and 3. Or, "All lines that start off parallel remain parallel" is no longer true generally since we no know of spherical geometry where lines that start off parallel will ultimately intersect.

Whatever disagreement exist between scientist about the truth of mathematics or science there is always, ultimately, one correct theory or explanation. Disagreements are overcome by the iterative process of mathematics and science as a result of which ever deeper and more general theorems, or ever better theories and explanations of natural phenomena, are derived always aiming towards the fundamental truths or descriptions of reality. None of these theorem or theories are dependent on social or historical context or the psychological state or philosophical beliefs of the person seeking the truth. In other words, these theories are the only ones which we would always ultimately have obtained.

An example of relativism in physics

As an example of the problems which can occur if we accede to social beliefs consider the case of Edwin Hubble and Albert Einstein. The following is taken from *The Culture of Science: The Nature of Scientific Knowledge*, Anthony Carpi, Anne E. Egger:

> In 1929 the American astronomer Edwin Hubble put forward the idea, based on data he had collected, that the universe was expanding. "Several years earlier, Einstein had published his general theory of relativity (Einstein, 1916). In formulating the theory, Einstein had encountered one significant problem: General relativity predicted that the universe had to be either contracting or expanding – it did not allow for a static universe. But a contracting or expanding

universe could not be eternal, while a static, non-moving universe could, *and the prevailing cultural belief at the time was that the universe was eternal.* Einstein was strongly influenced by his cultural surroundings. As a result, he invented a "fudge factor," which he called the cosmological constant, that would allow the theory of general relativity to be consistent with a static universe. But science is not a democracy or plutocracy; it is neither the most common or most popular conclusion that becomes accepted, but rather the conclusion that stands up to the test of evidence over time. Einstein's cosmological constant was being challenged by new evidence. [...] Upon seeing Hubble's work, even Albert Einstein changed his opinion of a static universe and called his insertion of the cosmological constant the "biggest blunder" of his professional career. Hubble's discovery actually confirmed Einstein's theory of general relativity, which predicts that the universe must be expanding or contracting. Einstein refused to accept this idea because of his cultural biases. His work had not predicted a static universe, but he assumed this must be the case given what he had grown up believing. When confronted with the data, he recognized that his earlier beliefs were flawed, and came to accept the findings of the science behind the idea. This is a hallmark of science: While an individual's beliefs may be biased by personal experience, the scientific enterprise works to collect data to allow for a more objective conclusion to be identified. Incorrect ideas may be upheld for some amount of time, but eventually the preponderance of evidence helps to lead us to correct these ideas. Once used as a term of disparagement, the "Big Bang" theory is now the leading explanation for the origin of the universe as we know it."

Commentary

Over the millennia mathematics has developed across different parts of the world: Babylon, Egypt, Arabia, China, India, Europe to name a few. Nowadays, mathematics has become standardised in terms of notation, rigour and process. The effect of this standardisation across the world is that everybody has to think in a particular mathematical way if any person from one part of the world is to understand the maths of a person from another part of the world.

But this was not the case in ancient times when communication was difficult or non-existent, as some countries were isolated from other countries. This then begs the question as to what type of mathematics was created in these individual isolated parts of the world. Did they each develop a radically different conception of what mathematics was and how it worked and how it was used? Was the mathematics of the ancient Chinese totally different to that of the Babylonians *such that it would be impossible to transform the mathematics of one civilisation into the mathematics of the other civilisation?*

For example, we understand numbers to be things that

- count, i.e. 1, 2, 3, 4, 5, 6, ...
- measure, i.e. use numbers to measure length, volume, mass, energy, etc...,
- group, in other words we have 3 apples, 7 bananas,

And we think of arithmetic to be the process which gives us things such as 1 + 1 = 2, or 2*3 = 3*2, etc. Does the concept of relativism imply that one culture would think of numbers and arithmetic differently, say 1+1=3? Yes it does. This means that one group of people has a particular conception of number and arithmetic, and the other group have a totally different conception of number and arithmetic, *neither of these conceptions being able to be reconciled*. In other words, there is no way one group's idea of number and process of arithmetic can be transformed into the other group's idea number and process of arithmetic. In the terminology of Thomas Kuhn (who wrote a book called *The structure of scientific revolution*) these two groups hold a conception of mathematics which is incommensurable (you can research what this means or you can wait until week 5 for what this means). There then exists a radically different conception as to what numbers are, what a length or area means, how to perform arithmetic and geometry and algebra, etc... This is the view of mathematical relativism.

But even if such a view was true one or both of these cultures would have to radically change their maths in order to account for numbers which are universally constants. Numbers such as

- mathematics: π, e (the exponential number), ϕ (the golden ratio),
- physics: G (Newton's constant of gravitation), c (speed of light in a vacuum), h (Planck's constant)

etc., would require both groups to hold the same underlying conception of number and arithmetic. They may have expressed their ideas differently and they may have operationalised the arithmetic differently but the results of mathematics between the two groups, at the fundamental level of what mathematics really is, would be the same.

Now, from exercise 1 and 2 on the first page of these notes it might seem as if mathematics (or at least arithmetic) is relative not absolute. We work using the decimal (base 10) number system but we could equally have chosen to work in binary (base 2) or any other base, and it would not have made a difference. This may be true. So is our choice to work in decimal just a social convention? Maybe. But underlying the infinite number of different bases we could work in (from binary to base 100 to base 1,000,000 and beyond) lies the idea of number and arithmetic. My belief is that whatever numbers and arithmetic we could have discovered, we discovered the one which allowed us to build an organised and consistent mathematics for the world we live in and, *importantly*, we discovered the universal constants of mathematics and physics.

Science aims to find the truth about the nature of physical phenomena: what they are, how they behave, etc. Mathematics (pure and applied) also aims to find truth. Scientific knowledge is cumulative: it builds on the work of other people; better data is collected; more accurate experiments are conducted; errors are corrected; more accurate explanations are developed; etc. But the fundamental principle or the essential idea behind the physical phenomena remains the same. Furthermore, *The convergence of multiple lines of evidence on a single explanation is what creates the solid foundation of scientific knowledge.* Different scientists approaching the problem from different perspectives but producing the same answers is also a factor which demonstrates that there is an absolute truth/existence to the things science studies.

I consider these truths absolute, in other words they do not change according to changing personal, societal or political values. A mathematical theorem once proved true is always true. A physical phenomenon such as gravity, aerodynamics or buoyancy remains true. If this were not the case the universe would not be stable. Gravity would change: sometimes a ball thrown in the air would return to Earth and sometimes it would go off into space; planes would sometimes stay in the air or would sometimes would never be able to take off, i.e. aerodynamics would be inconsistent; boats would sometimes float and sometimes sink, i.e. buoyancy would be inconsistent.

Notice that I am referring to the essential nature of physical phenomena not the mathematics or physics used to describe the phenomena. When it comes to mathematically describing such phenomena we are in the realm of modelling. Mathematical models are always up for revision and improvement, but the aim is always to get to the truth of how things work. In other words we are always iterating the quality of our mathematics and physics, and hence our models of natural phenomena towards ever better representation of true nature of the physical phenomena.

For me there is no such thing as relativism in science and maths. There is political pressure, there is societal pressure, there are personal biases towards what science is conducted and how it should be conducted. But ultimately when we find a scientific or mathematical theory that works then we have shown a degree of understanding of the truth about the physical phenomena under study. Political pressures, societal pressures, and/or personal biases from individual scientists or a community of scientists which influence discovery for the worse can/will be overcome over time.

We would always have ended up developing a correct mathematical theory of gravity, whether Newton's laws or Einstein's relativity theory; we would always have ended up discovering the principle of aerodynamics and the principle of buoyancy, and thus developing appropriate mathematics to describe these principles.

Continuing on this theme, suppose I choose to believe that I can defy gravity. Suppose I choose to believe that I can fly. So i go to the top of a building and jump off believing I will be able to fly. The truth is that I cannot fly because I, and human beings, are not designed to be able to do this. It is not that my belief is relative, namely that you are free to believe you cannot fly and I am free to believe that I can fly. It is that gravity and aerodynamics are truths of nature and work as they do, and that I am constrained by the design of my body to not be able to fly. If I ignore reality there are serious consequences to my life.

So, it is with mathematics. We can invent all sorts of mathematics which may or may not be absolute truths, and we will come to know the truth of our mathematics when we live with the consequences of it. For example, in the past people believed that $a \times b = b \times a$ where a and b are real numbers (and this is still true but only in a certain context). Then it was found that $A \times B \neq B \times A$ for two matrices A and B. Furthermore, for vectors (which are objects with magnitude and direction) the vector product is also not commutative: $\boldsymbol{a} \times \boldsymbol{b} \neq \boldsymbol{b} \times \boldsymbol{a}$. When people realised this they started developing all sorts of new algebras. Does this mean that mathematics is fundamentally relative? No. Matrices and the vector product are used in many areas of technology, science and engineering. So the underlying reality of physical phenomena can be modelled by these non-commutative mathematics. This means such a mathematics is valid. I contend that there must therefore be a mathematics underlying both commutative and non-commutative arithmetic (which we haven't discovered yet) which is the absolute mathematical truth which explains both commutative and non-commutative arithmetic.

But if I continue in my quest to invent new mathematics that has no foundation in absolute reality then I have divorced mathematics from reality and now I can speak of mathematical relativism. Then I can say that “anything goes” in mathematics.

Exercises

1) Identify the fundamental aspects of relativism and absolutism described above. How do these aspects apply to your discipline? To what extent/degree do these aspects apply? Can you find examples which illustrate (degrees of) relativism and/or absolutism in your discipline?

The following exercise will require some time to do, and unless you already know the topics you won’t be able to answer these straight away.

2) Consider your undergraduate degree. What aspects/theories of your discipline were considered acceptable in the past which are now considered wrong? Why were they considered acceptable? What beliefs did people of the time have about these aspect/theories which made them acceptable? You will need to look into the history of your discipline to answer this question. Some ideas are alchemy or phlogiston (from chemistry), or the concept of heat or force (from physics).

For the last two you might be interested in Max Jammer’s books: “The concept of heat” and “the concept of force” where he traces the historical development of these two concepts.

3) Choose a topic from your undergraduate degree. For example, if you studied some sort of mechanical engineering you might choose aerodynamics, hydrostatics, dynamics, buoyancy, thermodynamics, materials, etc. Now identify those aspect of the topic which are contingent on more fundamental aspects or principles (revise the idea of contingency by looking back to the example on arithmetic).

Are those aspects contingent on even deeper and more fundamental aspects? Is there an ultimate fundamental principal which is the essence of topic, and that could be called an absolute truth?

Or, is the development of your topic relative to some socially agreed standard of what is correct or acceptable?